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Abstract

We explore the consequences of assuming that bounded space-time subsets contain a finite number of degrees of freedom. A physically natural hypothesis is that this number is additive for spatially separated subsets. We show that this assumption conflicts with the Lorentz symmetry of Minkowski space since it implies that a conserved current determines the number of degrees of freedom. However, the entanglement across boundaries can lead to a subadditive property for the degrees of freedom of spatially separated sets. We show that this condition and the Poincare symmetry lead to the Bousso covariant entropy bound for Minkowski space.

I. INTRODUCTION

The quantum theory of fields suggests that an infinite number of degrees of freedom live in the bounded space-time regions. To obtain a finite dimensional Hilbert space one should consider energies bounded above by some value in addition to a finite volume ¹.

However, there are reasons to suspect that the description in terms of a quantum field theory (QFT) should break down below some distance scale. If there is a fundamental cutoff the number of degrees of freedom could turn out to be finite.

Finite Hilbert spaces are also a consequence of the picture coming from the physics of black holes. These have an associated entropy equal to $A/(4G)$, where A is the horizon area and G the Newton constant. If this number can be included in the second law of thermodynamics [3], the black hole entropy would represent the maximum entropy of any system capable of collapsing to form the black hole (assuming this later is stable [4]). Thought experiments of this kind have lead to the idea of the holographic entropy bound [5]. This means that the entropy of a system enclosed in a given approximately spherical surface of area A is less than $A/(4G)$. An appropriate generalized version of this bound to

¹A consistent definition of a Hilbert space of states assigned to some finite region is at odds with the Reeh-Slieder theorem which implies that the field operators in the space-time region generate the whole Hilbert space when acting on the vacuum [1]. Even considering a bounded energy the notion of a dimension for Hilbert space projectors assigned to space-time sets is absent in the axiomatic versions of quantum field theory [2].

arbitrary space-like surfaces and general space-times was proposed by Bousso in [6,7], where it was called the covariant entropy bound. This is as follows. Given a spatial codimension two surface Ω it is possible to construct four congruences of null geodesics orthogonal to Ω , two past and two future directed. Suppose that one of these null congruences orthogonal to Ω has non positive expansion θ at Ω . Then, call H the subset of the hypersurface generated by the congruence that has non positive expansion. The hypersurface H is called a light-sheet of Ω . The covariant entropy bound states that the entropy in H is less than $A(\Omega)/(4G)$.

The covariant bound should be regarded as tentative. However, there are no known reasonable counterexamples. Indeed, the bound can be shown to be true in the classical regime under certain conditions equivalent to a local cutoff in energy, and when the metric satisfies the Einstein equations [8]. This includes a vast set of physical situations.

Thus the entropy bounds would lead to a finite number of degrees of freedom proportional to the bounding area. This is in apparent conflict with the intuitive picture coming from a cutoff in space-time. If the dataset on a Cauchy surface is arbitrary, this later roughly suggests that an independent degree of freedom should be assigned to sets of the order of the cutoff scale on these surfaces, leading to a number of degrees of freedom that increases with the volume rather than the bounding area.

In this work we explore the consequences of assuming that a finite number of degrees of freedom $n(A)$ can be assigned to bounded sets A in space-time. The function $n(A)$ can not be defined in QFT. We define it here through a set of very general and physically motivated conditions.

We often use the term number of degrees of freedom in the sense of some logarithm of the Hilbert space dimension N , what is proportional to the statistical entropy $S = \log N$. For a system of independent spins $\log_2 N$ coincides with the number of spins ². The naive interpretation of the dimension $N(A)$ is that it represents the number of independent states that can be localized inside some Cauchy surface for A .

II. ORDER AND CAUSALITY

The function $n(A)$ is a real function of space-time subsets. We will assume that this function is an intrinsic property of the space. Evidently if a set A includes a set B , then it also contains all degrees of freedom that under some reasonable definition can be said to belong to B . Thus, an imperative condition for the function n is that it preserves the inclusion order between sets, that is

$$\text{I- Order} \qquad B \subseteq A \Rightarrow n(B) \leq n(A) \qquad (1)$$

for any A and B .

²It is not necessary to leave continuity at this point. The dimension N (and $\log(N)$) need not be discrete if we accept more general structures than Hilbert spaces. Continuous dimension functions can be given to some lattices of projectors of von Newman algebras. These would be type type II factors in contrast with the type I factors corresponding to Hilbert spaces and the type III factors that form the algebras of operators of bounded regions in algebraic quantum field theory [2].

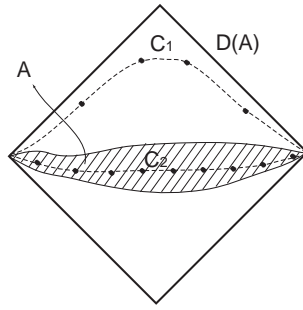


FIG. 1. The causal development $D(A)$ of the set A . The surface C_2 is a Cauchy surface for A while C_1 and C_2 are Cauchy surfaces for $D(A)$. The points on C_1 and C_2 represent discrete degrees of freedom on these surfaces (see the discussion at the end of Section III).

The causal structure imposes that the physics inside the whole causal development $D(A)$ of a set A must be described in terms of the same degrees of freedom. The causal development of a set A in space-time is the set of points through which any inextendible time-like curve intersects A . A typical $D(A)$ set has the shape of a diamond (see Fig.1). Thus, the data on the set A for wave like equations determines the variables in the whole set $D(A)$. This would imply that $n(D(A)) \leq n(A)$, but taking into account that $A \subseteq D(A)$ and (1) we have $n(D(A)) = n(A)$.

From now on we will take a more conservative domain for the function $n(A)$, and focus attention on a special class of sets. These are the ones that have a Cauchy surface³. We call a Cauchy surface for a set A to an achronal set C included in A such that $A \subseteq D(C)$. Thus, we will not consider the sets like two time-like displaced diamonds U_1 and U_2 . This is because it is possible that the function n can not be defined at all for these type of sets since the Cauchy problem is not well defined on them. Evidently the data on the Cauchy surfaces for U_1 and U_2 is necessary in order to solve a wave equation, but they are not independent.

The argument we have given for the validity of a causal law for n is in terms of degrees of freedom. The same reasoning can be made for the number of states. If a state can be localized inside $D(A)$, meaning in a Cauchy surface for $D(A)$, then its future or past will pass through A , and the converse is also true. This assumes that states cannot be created or destroyed along their evolution and orthogonal states remain orthogonal. Thus, underlying our second assumption is not only causality but unitarity. The total number of independent states that can be localized on the Cauchy surfaces C_1 and C_2 for the diamond shaped set in Fig.1 is the same. Then the second postulate reads

$$\text{II- Causality + Unitarity} \quad n(D(A)) = n(A). \quad (2)$$

With the interpretation of n as the statistical entropy $\log N$ it may look strange to assign this number to space-time subsets rather than to subsets of the phase space. However, in theories for wave equations the sets of the form $D(A)$ uniquely determine subsets of the

³The function D is sometimes defined only for sets having a Cauchy surface [9].

phase space through the initial data on their Cauchy surfaces ⁴. This is not the case for sets without a Cauchy surface.

Thus, up to now we have only asked n to be an order preserving function on the sets of the form $D(C)$ for some achronal set C . This is not very restrictive.

III. ADDITIVITY AND A NUMBER OF DEGREES OF FREEDOM PROPORTIONAL TO THE VOLUME

Degrees of freedom located in space-like separated regions would be independent since two operators based in them commute with each other. If we think in a kind of lattice of spins on a Cauchy surface, the number of independent spin degrees of freedom simply add for non intersecting subsets of the surface. States for a union of spatially separated regions can be formed by tensorial product, the dimension N then increasing as the product of dimensions. Based on this heuristic idea we can propose the following additivity postulate

$$\text{III(a)- Additivity} \quad A \text{ spacelike to } B \Rightarrow n(A \cup B) = n(A) + n(B). \quad (3)$$

Equation (3) makes the function n additive over subsets of a Cauchy surface C of a given causal diamond U . Thus, n can be written as the integral of a volume form $\omega_{C,x}$ on the surface C . We will focus on strictly space-like Cauchy surfaces. The form $\omega_{C,x}$ at a point x on C depends on the Cauchy surface C only locally, that is, it is independent of how the Cauchy surface is extended out a neighborhood of x on C . Thus, a better nomenclature for the forms giving place to the function n is $\omega_{C,x} = z(x, \eta^\mu) \sqrt{g} \eta^\mu \epsilon_{\mu\nu\alpha\beta}$, where η^μ , $\eta^\mu \eta_\mu = 1$, is the normalized future directed time-like vector orthogonal to C at x . To be explicit, we can represent n as

$$n(D(C)) = \int_C z(x, \eta^\mu(x)) \sqrt{g} \eta^\mu(x) \epsilon_{\mu\nu\alpha\beta}, \quad (4)$$

where C is any space-like Cauchy surface for $D(C)$.

The combination of causality and additivity imposes severe restrictions to the function $z(x, \eta^\mu(x))$. To see this let us think in a diamond set U of a differential size, so at zero order we can use the flat metric, and assume the function z is differentiable. Let us draw a Cauchy surface for U using pieces C_i of spatial planes passing through the diamond base faces, and let η_i be the corresponding normal vector to C_i . The number $n(U)$ must not depend on the chosen planes. Therefore, at zero order it is

$$\sum_i z(x, \eta_i) v(C_i) = \text{constant}, \quad (5)$$

where $v(C_i)$ is the volume of the piece of spatial plane C_i we used for constructing the Cauchy surface. Thus, this equation constrains the function $z(x, \eta)$ for fixed x and different

⁴At least for globally hyperbolic space-times this type of subsets coincides with the bounded causally closed sets [10]. Surprisingly, they form an orthomodular lattice, sharing this property with the lattice of physical propositions in quantum mechanics [11,12].

η . The result is that the function $z(x, \eta)$ must be of the form $j_\mu(x) \eta^\mu$ for a vector field j_μ . Then the integral in equation (4) is the flux of a current over the Cauchy surface. As the number $n(D(C))$ is independent of C the current j must be conserved.

Resuming, we have that

$$n(D(C)) = \int_C j_\mu^* \quad (6)$$

for a conserved j_μ ⁵.

Thus, it follows that the number of degrees of freedom comes from a conserved current. This can not be obtained in a general space-time from the metric alone. In particular it breaks the Lorentz symmetry of Minkowski space.

Assuming finite dimensions, we have that either additivity or unitarity is wrong, or Lorentz symmetry is broken. This seems to correlate with some possibilities to regularize quantum field theories. For finite regularization parameter the Pauli-Villars and the higher derivatives regularizations give non unitary theories, and a theory on the lattice is not Lorentz covariant.

The reason why additivity fails can be seen more easily looking at Fig.1. Suppose we have a covariant way of assigning degrees of freedom to the Cauchy surfaces C_1 and C_2 , for example taking the independent spin degrees of freedom to be separated by some spatial distance. Then moving C_1 to approach the null boundary of the causal diamond its volume goes to zero reducing the number of independent spins. As C_1 and C_2 must have the same number of degrees of freedom since they describe the same physics, most of the spins on C_2 must not be independent, and the prescription of separating the independent spins by a fixed distance is wrong. However, the degrees of freedom on the spatial corner may well turn out to be independent when restricting attention to a particular diamond (a related version of these ideas is in Ref. [14]).

IV. SUBADDITIVITY AND THE BOUSSO COVARIANT ENTROPY BOUND

Therefore if we want to keep the postulates of Section II, and not to break Lorentz invariance, we have to give up additivity.

Given two pieces C_1 and C_2 of a Cauchy surface C such that $C_1 \cup C_2$ covers C , it is possible that some degrees of freedom are shared between the C_1 and C_2 , and then the number $n(C)$ will be less than the sum $n(C_1) + n(C_2)$, while the opposite would be more difficult to justify if the knowledge of the physics in C_1 and C_2 must determine the physics in C . Similarly, the entanglement of states for adjacent regions in a Cauchy surface can lead to double counting the same states [15]. The states on the boundary is counted by one or by both, the states restricted to C_1 and the ones restricted to C_2 . Otherwise some states would escape from C . If $N(C_1) \cdot N(C_2) \geq N(C)$ then $n(C_1) + n(C_2) \geq n(C)$ would hold for

⁵Equation (3) makes the function n a state on the lattice of causally closed sets [11,12]. A full demonstration that a conserved current defines a state on the lattice of causally closed sets though the formula (6) can be consulted in [13].

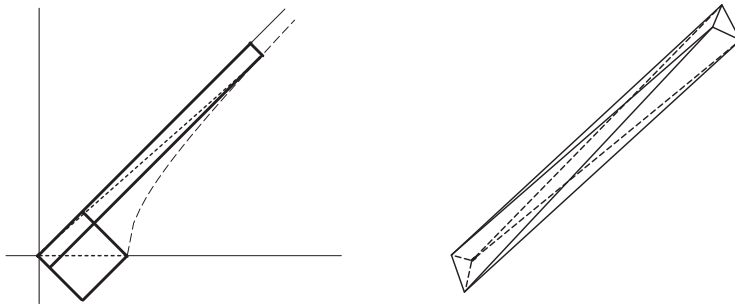


FIG. 2. Left: boosting a reference diamond. The short dashed line represents the Cauchy surface formed by the diamond base. Right: a view of a three-dimensional stretched reference diamond.

the entropies. What would be surprising is that such entanglement entropy could turn out to be relevant.

Let us give place to the entanglement and assume that a subadditive law is valid. If two sets U_1 and U_2 cover a Cauchy surface for U then

$$\text{III(b)- Subadditivity} \quad U \subseteq D(U_1 \cup U_2) \Rightarrow n(U) \leq n(U_1) + n(U_2) . \quad (7)$$

All requirements for the function n up to now are conformal invariant. In fact, they only depend on the causal structure rather than the conformal one [16]. For example postulate (2) makes n trivial for some spaces without achronal sets.

Let us see more closely the implications of (1), (2) and (7) for the case of Minkowski space. The Poincare symmetry is not shared by conformal transformations of the flat metric. Thus, imposing it will select the Minkowski metric. This symmetry has very strong implications for our function n . To see this let us take a small reference diamond U with a cubic base on the surface $t = 0$ in a given reference frame and with faces of area α . With a boost it is possible to stretch the diamond along a null direction as in Fig.2. This stretched diamond must then have the same number of degrees of freedom as the original one. The boosts do not change the size of the transversal dimensions, and the area of the diamond base spatial faces is of course unchanged. The null line that goes from the diamond tip to the central point in the cubic face is orthogonal to that face.

Consider now a space-like surface Ω of codimension 2 and such that its orthogonal null congruence has negative expansion at Ω . With many copies of boosted reference diamonds, using now rotations and translations to align their spatial faces along the spatial surface Ω , we can cover the whole light sheet of Ω , as shown in Fig.3(a). The number of reference diamonds needed is the area of Ω in units of α . Each stretched diamond has the same number of degrees of freedom, $n(U)$. Then, using the subadditive property we have

$$n(H) \leq \frac{n(U)}{\alpha} a(\Omega), \quad (8)$$

where H is the light sheet of Ω , and $a(\Omega)$ is the area of the $(d-2)$ dimensional spatial surface Ω . Note that the fraction on the right hand side is independent of Ω as long as this surface is big enough. As shown in Fig.3(b), the same construction does not work for expanding light sheets because the stretched diamonds separate each other along the null congruence, and their transversal size can not increase since boosts leave transversal dimensions unchanged.

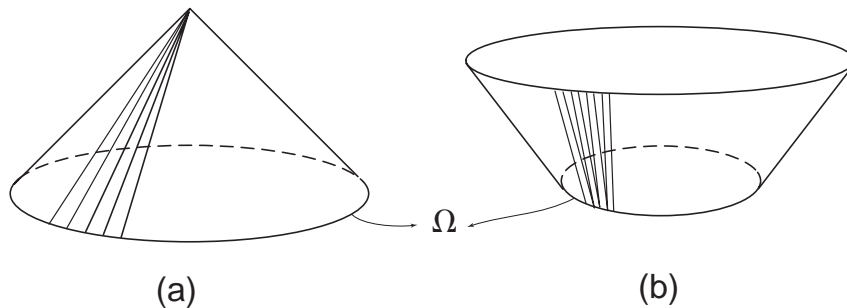


FIG. 3. (a) Covering a light-sheet with boosted reference diamonds. The picture only shows their triangular null faces (in $d = 3$). (b) It is not possible to cover an expanding surface with stretched reference diamonds since basing them on the surface Ω and aligning them along the null lines they separate each other while their transversal size is constant.

Therefore, we recover all the elements of the Bousso entropy bound from these simple assumptions. The constant relating area to entropy is not determined at this point.

de Sitter space is conformally flat and it is also maximally symmetric with symmetry group $O(4, 1)$. Then we can use this symmetry group to emulate what we have done for Minkowski space. To keep explicit the symmetry we can work in $4+1$ dimensional Minkowski space, where de Sitter space is given by the hypersurface $x_0^2 - (x_1^2 + x_2^2 + x_3^2 + x_4^2) = -\Lambda^2$. The Lorentz group $O(4, 1)$ manifestly lives this surface invariant. Let us consider the spatial surface $x_0 = l$, and the sphere Ω given by $(x_1^2 + x_2^2 + x_3^2) = r^2$, or equivalently $x_4^2 = l^2 + \Lambda^2 - r^2$, on it. Among all the null congruences orthogonal to the 2-sphere Ω in 5 dimensions there are 4 included in de Sitter space. These are also the 4 null congruences orthogonal to Ω in de Sitter space. Then, we can use the Lorentz boosts in 5 dimensions as we did before to stretch a small 4 dimensional diamond covering a piece of Ω along the null congruence orthogonal to it. These boosts must be in a direction orthogonal to Ω but parallel to the hyperboloid at the point where the small reference diamond is seated. Using rotations we can transport diamonds along the spatial surface $x_0 = l$. To transport the reference diamond along the x_0 direction we have to use the boosts in direction orthogonal to the hyperboloid at the point where the diamond is located. The areas are unchanged by symmetry transformations and we obtain again the bound (8). As happens in the case of Minkowski space, this procedure can work to obtain a bound as long as the surface is contracting. For positive l there is one future directed and one past directed light sheet of Ω for $r < \Lambda$, while for $r > \Lambda$ there are two past directed light-sheets.

For spaces with less number of symmetries it would not be possible to obtain the covariant bound using a single reference diamond as we have done here. In the general case, given a causal structure we have to study the possible solutions to the set of constraints (1), (2) and (7). We remark that this system of inequalities is linear, thus linear combinations with positive coefficients form also solutions. We will not develop further in this sense here. Instead let us go back to the case of Minkowski space.

V. THE SCALING NEWTON CONSTANT

The equation (8) compared with the covariant entropy bound suggests the interpretation of $\frac{4n(U)}{\alpha}$ as G^{-1} . However, the value of $\frac{n(U)}{\alpha}$ must be used only for the light-sheets H with $a(\Omega)$ greater than α . The smallest value of $\frac{n(U)}{\alpha}$ for the U adequated to a given H is what should be used to have the strongest bound.

The construction of the preceding Section can be generalized to investigate the scaling properties of $n(U)$ in a way similar to the renormalization group, but where we have inequalities instead of equations. The order and subadditivity play opposite rolls in the inequalities, and the Poincare symmetry is the tool to use for comparing different sets. The equation (8) is obtained by measuring a set using another one and is a special case of these inequalities. Many other configurations could be used to provide different inequalities of this kind.

For example consider diamonds with spatial rectangular base at $t = 0$ in a given reference frame. Let us call them $U_{(l_1, \dots, l_{d-1})}$ where (l_1, \dots, l_{d-1}) are the side lengths. Here d is the space-time dimension. The diamond $U_{(l_1, \dots, l_{d-1})}$ can be measured by two copies of the diamonds $U_{(l_1, \dots, \varepsilon)}^1, \dots, U_{(l_1, \dots, l_{k-1}, \varepsilon, \dots, l_{d-1})}^k, \dots, U_{(\varepsilon, \dots, l_{d-1})}^{d-1}$, where ε is smaller than all the other sides. It is enough to cover with the big faces of the $U_{(l_1, \dots, l_{k-1}, \varepsilon, \dots, l_{d-1})}^k$ the corresponding faces of $U_{(l_1, \dots, l_{d-1})}$ and make the boosts in the small direction (the k^{th} direction). The subadditive and order properties give

$$\begin{aligned} \max(n(U_{(l_1, \dots, l_{k-1}, \varepsilon, \dots, l_{d-1})}^k)) &\leq n(U_{(l_1, \dots, l_{d-1})}) \leq 2(\sum n(U_{(l_1, \dots, l_{k-1}, \varepsilon, \dots, l_{d-1})}^k)) \\ &\leq 2(d-1) \max(n(U_{(l_1, \dots, l_{k-1}, \varepsilon, \dots, l_{d-1})}^k)) \end{aligned} \quad (9)$$

There we see that $n(U_{(l_1, \dots, l_{d-1})})$ is roughly given by the function n of its facial diamonds.

Let us choose $l_k = l$ smaller than all the other sides, which makes the face perpendicular to this side the biggest one. Then we see from (9) that the function $n(U_{(l_1, \dots, l_{k-1}, \varepsilon, \dots, l_{d-1})}^k)$ as ε goes to zero is bounded below by $n(U_{(l_1, \dots, l_{k-1}, l, \dots, l_{d-1})})/(2(d-1))$. It also decreases with ε and thus must converge. We call to this limit for facial sets

$$n_{d-2}(l_1, \dots, l_{d-2}) = \lim_{\varepsilon \rightarrow 0} n(U_{(l_1, \dots, l_{d-2}, \varepsilon)}). \quad (10)$$

We have only rotations and translations as symmetries for the facial sets, since we have run out of boosts. The function n_{d-2} is ordered by inclusion. The subadditive law implies subadditivity for faces, that is, the sum of the n_{d-2} of faces that cover a given face must be greater than the function n_{d-2} for this face.

Covering with copies of a face (r_1, \dots, r_{d-2}) another face (l_1, \dots, l_{d-2}) with multiple side lengths, $l_k = m_k r_k$, $m_k \geq 1$ integer, we obtain

$$\frac{n_{d-2}(l_1, \dots, l_{d-2})}{l_1 \dots l_{d-2}} \leq \frac{n_{d-2}(r_1, \dots, r_{d-2})}{r_1 \dots r_{d-2}}, \quad (11)$$

$$n_{d-2}(r_1, \dots, r_{d-2}) \leq n_{d-2}(l_1, \dots, l_{d-2}). \quad (12)$$

Thus, the $(d-2)$ dimensional function $\tilde{G}_{d-2}^{-1} = (4n_{d-2}/A_{d-2})$ is a decreasing function in the sense of the inclusion order, and $(A_{d-2} \cdot \tilde{G}_{d-2}^{-1})$ is an increasing function. As a result the behavior of \tilde{G}_{d-2}^{-1} is bounded between a constant and a constant times A_{d-2}^{-1} .

The area law for entropy appears here as a possibility unrelated a priori to gravity. The interpretation of \tilde{G}_{d-2} as proportional to the Newton constant is supported by the work [17]. There the entropy was taken proportional to the area and the stress tensor was interpreted in terms of a heat flux across null surfaces. Using the second law of thermodynamics as seen from accelerated observers to relate the changes of area with the heat flux it was shown that the Einstein equations for the metric can be obtained. It also follows the proportionality factor $1/4$ between $G^{-1} \cdot \text{area}$ and entropy. We think that a rewriting of that work in terms of the function n and where the stress tensor provides additional information to the inequalities could clarify the roll of gravity in the present context.

We note two things. The first is that according to (9) the ratio $\frac{n(U)}{A(\Omega_U)}$, where Ω_U is the surface forming the spatial corner of U , is of the order of \tilde{G}_{d-2}^{-1} for an area of the same size. The second is that \tilde{G}_{d-2}^{-1} can depend on the size and form of the surface. Gravity could be stronger at larger size according to (11).

We will leave a more complete analysis of the possible solutions of the system of inequalities for a future work. But we note that there are two solutions for n that are extremal in the sense explained before. These are $n(U) = c_1$ for a constant c_1 and $n(U) = c_2 A(\Omega_U)$. In this later case we have to consider only the sets U whose null border has positive expansion at Ω_U . A linear combination of solutions with possitive coefficients is also a solution, so $n(U) = c_1 + c_2 A(\Omega_U)$ is a solution. In this case we have

$$\tilde{G}_{d-2} = \frac{1}{4(c_2 + \frac{c_1}{A_{d-2}})}. \quad (13)$$

The case of $c_2 = 0$ corresponds to a number of degrees of freedom that saturates and does not increase any more with the size. All sets share the same degrees of freedom and the entanglement is total. The case $c_1 = 0$ is when the Newton constant is really constant and the area law for the entropy is valid.

In general both types of solutions can contribute, and then the picture that emerges is that as we go to greater sizes the term corresponding to a number of degrees of freedom proportional to the area dominates and \tilde{G}_{d-2} is constant. This is a standard gravity with constant G . At lower sizes the constant number of degrees of freedom could dominate and the Newton constant would vanish. The Plank scale would increase as we go to smaller scales making impossible to surpass it. This limit is conformally invariant since the number of degrees of freedom is constant.

In a general space-time n has to be assigned to diamonds U , and being subadditive it is less than the sum of a number proportional to the spatial corner area of small stretched diamonds that cover the null border of U . Thus we see a reason why n would be an additive function on the spatial border of U for big sets. This is because it is the most relevant term in the language of the renormalization group since all other terms have to be subadditive. The area is the less suppressed of the additive terms on the spatial corner when the curvature is small. Thus, it is possible to imagine that given a causal structure the solution of the inequalities would give an area law, from which to extract the metric. This later would result composed by two parts, a net of causal diamonds corresponding to some lattice of projectors, and the function n , corresponding to the logarithm of a dimension function for the projectors. The metric would have no sense for smaller diamonds with entropies that are non additive over the spatial corner.

However, as the basic element is the causal structure and not the metric, it seems that a different interpretation for the conformal case $n = \text{constant}$ is possible. This implies that the symmetry under scaling of the causal structure of Minkowski space is realized. Thus, it is possible that a different metric could realize this symmetry in the sizes where n is constant. A sector of de Sitter space with the metric

$$ds^2 = \frac{\Lambda^2}{t^2}(dt^2 - dx^2 - dy^2 - dz^2) \tag{14}$$

is invariant under scaling. Time translation symmetry is however broken.

VI. ACKNOWLEDGMENTS

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